Core Mathematics C3 For Edexcel Advanced Level

Paper H
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. Prove by contradiction that there are no pairs of positive integers x and y such that

$$x^2 - y^2 = 18. \qquad \textbf{(6)}$$

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2. Simplify $\dfrac{2x^2 + 5x - 3}{4x^2 - 1} \times \dfrac{2x^2 + x}{x^2 + 8x + 15}$. **(6)**

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3. The function f is defined by

$$\text{f}: x \mapsto 2x - 1, \quad x \in \mathbb{R}.$$

(*a*) Find $\text{f}^{-1}(x)$ and state the domain of f^{-1}. **(2)**

The function g is defined by

$$\text{g}: x \mapsto 3x^2 + 1, \quad x \in \mathbb{R}.$$

(*b*) Find $\text{gf}^{-1}(x)$. **(2)**

(*c*) Find the values of x which satisfy $\text{gf}^{-1}(x) = \dfrac{7}{4}$. **(3)**

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4. Given that $x = \cos^{-1}\left(\dfrac{-5}{13}\right)$, find the exact value of

(*a*) $\tan x$, **(4)**

(*b*) $\text{cosec}\, 2x$. **(4)**

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5. (*a*) Given $0 < a < b$, sketch the graphs of $y = x - a$ and $y = |x - b|$ on the same axes. **(4)**

(*b*) Find the area of the triangle enclosed by the two graphs and the y-axis. **(4)**

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6. Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^y = 3 - x$$

$$2y = \ln(x + 2)^2 - 4. \qquad \textbf{(8)}$$

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7. (*a*) Differentiate the following with respect to x, simplifying your answers.

(i) $x \ln(3x + 4)$ **(3)**

(ii) $(3x^2 + 7)^5$ **(3)**

(*b*) Using the derivatives for $\sin x$ and $\cos x$, prove that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. **(4)**

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8. The curve with equation $y = \mathrm{e}^{-\frac{1}{2}x}$ crosses the y axis at the point $P(0, p)$.

(*a*) Sketch the graph of $y = \mathrm{e}^{-\frac{1}{2}x}$, showing the exact value of p. **(2)**

The normal to the curve at the point Q, with x-coordinate q, passes through the origin.

(*b*) Show that $x = q$ is a solution of the equation $2x\mathrm{e}^x - 1 = 0$. **(4)**

(*c*) Show that the equation in (*b*) can be rearranged in the form $x = \dfrac{1}{2\mathrm{e}^x}$. **(1)**

(*d*) Use the iteration formula

$$x_{n+1} = \frac{1}{2\mathrm{e}^{x_n}}, \quad \text{with } x_0 = 0.4,$$

to find x_1, x_2, x_3 and x_4. Hence write down, to 3 decimal places, an approximation for q. **(3)**

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9. (*a*) (i) Express

$$40\cos\theta - 9\sin\theta \qquad \text{in the form}$$

$$R\cos(\theta + \alpha) \qquad \text{where} \qquad R > 0 \qquad \text{and} \qquad 0 < \alpha < 90°.$$ **(4)**

(ii) Hence solve the equation

$$40\cos\theta - 9\sin\theta = 4$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place. **(3)**

(*b*) Solve the equation

$$6\sin\theta = 2\operatorname{cosec}\theta + 1$$

for $0 < \theta < 180$, giving your answers to 1 decimal place. **(5)**

END **TOTAL 75 MARKS**